

The random-anisotropy model in the strong-anisotropy limit

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Abstract. We investigate the nature of the critical behaviour of the random-anisotropy Heisenberg model (RAM), which describes a magnetic system with random uniaxial single-site anisotropy, such as some amorphous alloys of rare earths and transition metals. In particular, we consider the strong-anisotropy limit (SRAM), in which the Hamiltonian can be rewritten as the one of an Ising spin-glass model with correlated bond disorder: $\mathcal{H} = -J \sum_{\langle xy \rangle} j_{xy} \sigma_x \sigma_y$, where $j_{xy} = \vec{u}_x \cdot \vec{u}_y$ and \vec{u}_x is a random three-component unit vector. We performed Monte Carlo simulations of the SRAM on simple cubic L^3 lattices, up to $L = 30$, measuring correlation functions of the replica-replica overlap, which is the order parameter at a glass transition. The corresponding results show critical behaviour and finite-size scaling. They provide evidence of a finite-temperature continuous transition with critical exponents $\eta_o = -0.24(4)$ and $\nu_o = 2.4(6)$. These results are close to the corresponding estimates that have been obtained in the usual Ising spin-glass model with uncorrelated bond disorder, suggesting that the two models belong to the same universality class. This is consistent with arguments that suggest that the disorder correlations present in the SRAM are irrelevant.

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1. Introduction

Extensive theoretical and experimental work has been devoted to the study of amorphous alloys of rare earths and transition metals, for instance TbFe₂ and YFe₂. They are modeled [1] by a Heisenberg Hamiltonian with random uniaxial single-site anisotropy defined on a simple cubic lattice, or, in short, by the random-anisotropy model (RAM)

$$\mathcal{H} = -J \sum_{\langle xy \rangle} \vec{s}_x \cdot \vec{s}_y - D \sum_x (\vec{u}_x \cdot \vec{s}_x)^2, \quad (1)$$

where \vec{s}_x is a three-component spin variable, \vec{u}_x is a unit vector describing the local (spatially uncorrelated) random anisotropy, and D the anisotropy strength. In amorphous alloys the distribution of \vec{u}_x is isotropic, since, in the absence of crystalline order, there is no preferred direction.

Random anisotropy is a relevant perturbation of the pure Heisenberg model, the crossover exponent being [2] $\phi_D = 0.412(3)$. Therefore, random-anisotropy systems show a behavior that is different from that observed in pure Heisenberg systems. Even though the critical behavior of the RAM has been investigated at length in the last thirty years (see [3] for a review) we do not have yet a satisfactory picture of its critical behaviour. The Imry-Ma argument [4] forbids the presence of a low-temperature phase with nonvanishing magnetization for $d < 4$. However, this does not exclude the appearance of a glass transition with a low-temperature phase characterized by quasi-long-range order (QLRO), i.e., a phase in which correlation functions decay algebraically [5]. Functional renormalization-group calculations [6, 7] indicate that QLRO may set in for small values of D , in agreement with a Landau-Ginzburg calculation [8] of the equation of state for $D \rightarrow 0$. In the opposite limit $D \rightarrow \infty$ the RAM Hamiltonian reduces to that of an Ising spin glass with a correlated bond distribution. Indeed, for $D \rightarrow \infty$ one can write $\vec{s}_x = \sigma_x \vec{u}_x$, where $\sigma_x = \pm 1$ is an Ising spin, and obtain the Hamiltonian

$$\mathcal{H} = - \sum_{\langle xy \rangle} j_{xy} \sigma_x \sigma_y, \quad j_{xy} \equiv \vec{u}_x \cdot \vec{u}_y. \quad (2)$$

We call this model strong random-anisotropy model (SRAM) (We set $J = 1$ without loss of generality). Model (2) differs from the usual Ising spin glass model (ISGM) in the bond distribution: in Hamiltonian (2) the random variables j_{xy} on different lattice links are correlated. For instance, one has $\overline{\prod_{\square} j_{xy}} = 1/27$, where the product is over the links belonging to a given plaquette and the average is taken with respect to the distribution of the vectors \vec{u}_x .

An interesting hypothesis, originally put forward in [9], is that the SRAM transition is in the same universality class as that of the ISGM. This conjecture looks very plausible, since the SRAM is nothing but an Ising model with local disorder and frustration. In some sense we can think of the SRAM and of the ISGM as two different versions of the same model: in the SRAM disorder is associated with lattice sites, while in the ISGM disorder is associated with lattice bonds. They are analogous to the site-diluted and bond-diluted Ising model [10, 11], whose Hamiltonian is given by (2) with $j_{xy} = r_x r_y$ (site dilution) and $j_{xy} = r_{xy}$ (bond dilution), r being a random variable such that $r = 1$ ($r = 0$) with probability p (resp. $1 - p$). Note that the site-diluted model can also be interpreted as a bond-diluted model with a correlated bond distribution, exactly as is the case for the SRAM. Nonetheless, there is little doubt—though a satisfactory numerical check is still missing—that the two models belong to the same universality class.

Note that the SRAM is less frustrated than the standard ISGM, since in the SRAM $\overline{\prod_{\square} j_{xy}} = 1/27$. This fact does not rule out the conjecture since it is known that maximal

frustration is not necessary to obtain glassy behaviour. For instance, the random-bond Ising model with $j_{xy} = +1$ with probability p and $j_{xy} = -1$ with probability $1 - p$ has a glassy low-temperature phase for [12] $0.233 \lesssim p \lesssim 0.767$.

The identity of the SRAM and ISGM universality classes was confirmed in two dimensions by a renormalization-group calculation using the large-cell method [13], though in this case the critical point is at $T = 0$. In three dimensions instead, the critical behaviour of the SRAM has been controversial for a long time. While, for small D , numerical simulations [14, 15, 16, 17, 18] provided some evidence of the existence of a finite-temperature transition (though QLRO was never observed), in the SRAM even the existence of a finite-temperature transition was in doubt [18].

In [19] we study again the SRAM and find good evidence for the existence of a finite-temperature transition. The corresponding critical behaviour turns out to be compatible with the conjecture of [9]. Note that at the transition only the overlap variables, which are the usual order parameters at a spin-glass transition, become critical. Magnetic quantities do not show any critical behaviour, though we expect nonanalyticities induced by the critical modes. Thus, on both sides of the transition the system is paramagnetic. It is unknown whether this paramagnetic phase survives up to $T = 0$.

Note that our results predict an ISGM transition also in a generalized SRAM in which the vectors \vec{u} are N dimensional, for $N > 3$. Indeed, as N increases, the bond correlation decreases and, for $N \rightarrow \infty$, one reobtains the ISGM although with a nonstandard bond distribution. For $N = 2$ instead a *ferromagnetic* phase transition would be possible since the system is less frustrated than the case we have considered. Note that such a transition would only be observed in correlations of $\epsilon_x \sigma_x$, where the Ising variables ϵ_x should be chosen such as to have the couplings $\epsilon_x \epsilon_y j_{xy}$ ferromagnetic on a maximal set of lattice links. This ferromagnetic transition would not violate the Imry-Ma argument, since order in the σ_x variables does not imply order in the continuous variables \vec{s}_x . Most likely, as for $N = 3$, the low-temperature phase would still be paramagnetic even if some Ising variables magnetize.

A question that remains open is the behaviour of the RAM for finite anisotropy D . If there is indeed a low-temperature phase with QLRO for small D as predicted in [6, 7, 8], then there should be a critical value D^* such that ISGM behaviour is observed only for $D > D^*$. Nothing is known about D^* and we cannot even exclude that $D^* = \infty$, so that ISGM behaviour is observed only for model (2).

2. Results

In Ref. [19] we study the critical behaviour of the SRAM by means of Monte Carlo (MC) simulations. Since the model is essentially a spin glass we focus on the so-called overlap variables $\sigma_x \tau_x$, where σ_x and τ_x are associated with two different replicas of the model with the same bond variables. For the SRAM one can also consider the standard magnetic variables $\vec{s}_x = \sigma_x \vec{u}_x$. We find, in agreement with the results of [18], that these quantities are not critical, i.e. on the low-temperature side of the transition the system

is still paramagnetic. We study the behaviour of the SRAM in the high-temperature phase. This reduces the algorithmic problems—the MC algorithm becomes very slow as temperature is reduced—and allows us to consider lattices of size L^3 up to $L = 30$. For the MC dynamics, we combine the Metropolis algorithm with the random-exchange (parallel-tempering) method [20, 21].

In order to verify whether the system has a critical behaviour, we looked for the occurrence of finite-size scaling (FSS). For this purpose, we considered the ratios $R_A \equiv A(\beta, sL)/A(\beta, L)$ with $A = \chi, \xi$ (both quantities are associated with the two-point function of the overlap variables), fixing $s = 3/2$. If FSS holds, as L increases all points should eventually fall on a universal curve depending on $\xi(\beta, L)/L$. Our results, reported in Figures 1, 2, show that this happens: The data corresponding to the pairs $L = 16, 24$ are only slightly different from those with $L = 20, 30$, the difference being much less than that observed by comparing data with $L = 16, 24$ and $L = 8, 12$. Therefore, the data strongly suggest that FSS holds, (though, for $L \lesssim 30$, scaling corrections are significant) and therefore, that the system becomes eventually critical. At the critical point $R_\xi(\beta_c, L) = s$. Looking at Fig. 2 we see that all MC data satisfy $R_\xi(\beta, L) \lesssim 3/2 = s$, which indicates that they all lie in the high-temperature phase. This allows us to set a lower bound on the position of the critical point, $\beta_c \gtrsim 1.1$.

In order to determine the critical properties of the model, i.e. critical point and critical exponents, we used the iterative method which was introduced in [22] and generalized in [23] to include scaling corrections. It allowed us to obtain infinite-volume estimates of χ and ξ up to $\xi_\infty \approx 20$ (ξ_∞ is the infinite-volume second-moment correlation length) in the high-temperature phase. The starting point is the FSS relation

$$\frac{A(\beta, sL)}{A(\beta, L)} = F_A(s, \xi(\beta, L)/L) + L^{-\omega} G_A(s, \xi(\beta, L)/L), \quad (3)$$

valid for any long-distance quantity. Here s is an arbitrary number (in our calculations we fixed $s = 3/2$), $F_A(s, z)$ and $G_A(s, z)$ are universal scaling functions, and ω is a to-be-determined correction-to-scaling exponent. By iterating (3) it is possible to extrapolate the finite-volume estimates $A(\beta, L)$, obtaining the infinite-volume value $A_\infty(\beta)$.

Our results show that $\xi_\infty(\beta)$ increases quite rapidly in the range we have studied, $0.80 \lesssim \beta \lesssim 1.0$, confirming that the system eventually becomes critical. Fitting the infinite-volume results, we are able to determine the critical point and critical exponents associated with the critical behaviour of the overlap observables. We obtain:

$$\begin{aligned} \beta_c &= 1.08(4) \\ \eta_o &= -0.24(4) \\ \nu_o &= 2.4(6) \\ \gamma_o &= 5.3(1.3). \end{aligned} \quad (4)$$

The critical exponents are defined by $\chi \sim \xi^{2-\eta_o}$ and $\xi \sim (\beta_c - \beta)^{-\nu_o}$. The suffix o is introduced to remind that they refer to the overlap variables and not to the magnetic ones. In the analysis it is crucial to include corrections to scaling in the FSS (corrections

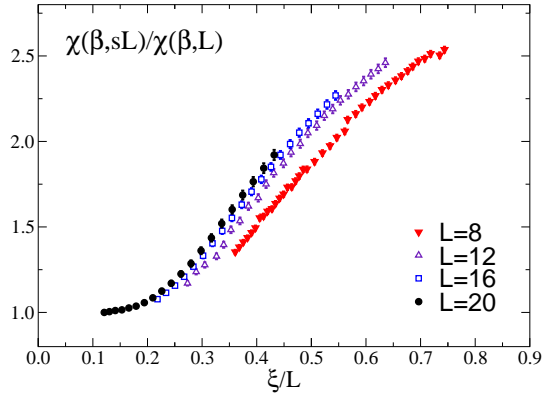


Figure 1. The FSS curve of the susceptibility χ for $s = 3/2$.

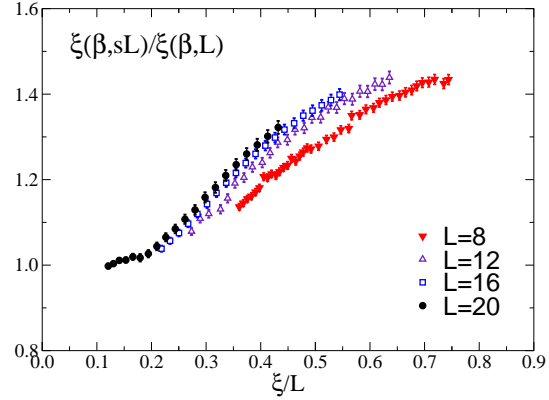


Figure 2. The FSS curve of the correlation length ξ for $s = 3/2$.

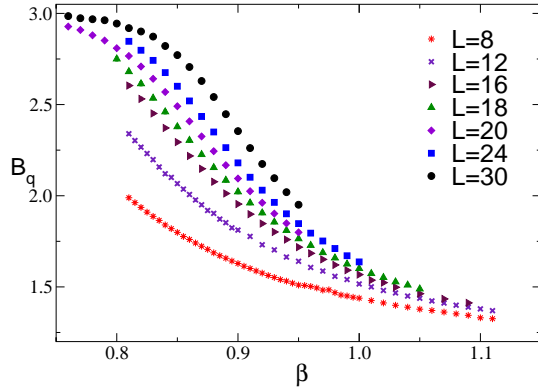


Figure 3. The quartic cumulant B_q of the overlap parameter for several lattice sizes.

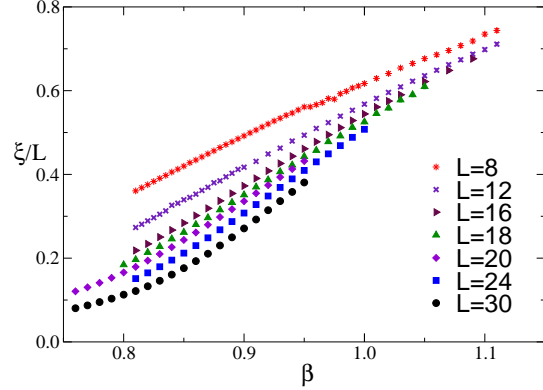


Figure 4. The ratio ξ/L for several lattice sizes.

behave as $L^{-\omega}$) and in fits of infinite-volume quantities (corrections behave as $\xi_\infty^{-\omega}$ as $\xi_\infty \rightarrow \infty$). The exponent ω was determined by studying the critical behaviour of two universal ratios that involve the four-point and the two-point correlation function of the overlap variables. We obtained $\omega = 1.0 \pm 0.4$.

Estimates (4) are close to those obtained for the ISGM and thus support the conjecture that the SRAM transition is in the same universality class as that of the ISGM. Ref. [24] quotes $\nu_o = 2.22(15)$, $\eta_o = -0.349(18)$, while [25] reports $\nu_o = 2.39(5)$ and $\eta_o = -0.395(17)$. Other estimates are reported in Table 1 of [25]. With the quoted error bars there is a small discrepancy between our estimate of η_o and those of [24, 25]. This difference should not be taken too seriously, since there are similar discrepancies among the estimates obtained by different groups for the bimodal ISGM, see Table 1 of [25].

In MC simulations the critical point is often determined by considering the crossing point β_{cross} of the Binder cumulant $B_q \equiv \overline{\mu_4}/\overline{\mu_2}^2$, where $\mu_k \equiv \langle (\sum_x q_x)^k \rangle$, or of the ratio ξ/L : indeed, $\beta_{\text{cross}} \rightarrow \beta_c$ as $L \rightarrow \infty$. Our data for B_q and ξ/L , which are reported in Figures 3 and 4, are compatible with $\beta_{\text{cross}} \approx 1.08$. Indeed, the estimates of both B_q

and ξ/L at fixed L get closer as β increases towards 1.08, although a crossing point is not clearly observed for those values of L ($L \leq 16$) that extend up to $\beta = 1.10$. This is probably due to scaling corrections that are particularly large in the SRAM.

We can perform a more quantitative check by using the results of [25] for the critical-point values B_q^* and $(\xi/L)^*$. They quote: $B_q^* = 1.475(6)$ (bimodal distribution) and $B_q^* = 1.480(14)$ (Gaussian distribution); $(\xi/L)^* = 0.627(4)$ (bimodal distribution) and $(\xi/L)^* = 0.635(9)$ (Gaussian distribution). These results are compatible with ours for B_q and ξ/L close to $\beta = 1.08$. For $\beta = 1.07$ we have $B_q = 1.411(4)$ ($L = 12$), $B_q = 1.434(6)$ ($L = 16$), and $\xi/L = 0.662(4)$ ($L = 12$), $\xi/L = 0.648(4)$ ($L = 16$). These results are very close to the ISGM estimates and show the correct trend. They are therefore consistent with the existence of a critical point at $\beta = 1.08 \pm 0.04$ and with the conjecture that the ISGM and the SRAM belong to the same universality class.

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